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Estimation of Future Discretionary Benefits (FDB) in Traditional Life Insurance

based on joint work with:
F. Gach, E. Kienbacher, G. Schachinger

Simon Hochgerner

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Contents

Market consistent valuation: terminology

- Life insurance with profit participation;
- Profit participation in the traditional sense: gross surplus is shared between policy holder and share holder (and tax office);
- Market consistent valuation: technical provisions according to Solvency II
- Technical provisions are sum of: best estimate and risk margin;
- Best estimate is sum of: guaranteed benefits (GB) and future discretionary benefits (FDB);

Market consistent valuation à la Solvency II

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Best Estimate Life w PP	203.1 bEUR
GB	163.6 bEUR
FDB	39.5 bEUR
Liabilities	234.5 bEUR
Assets	269.0 bEUR
Own funds (OF)	34.1 bEUR
Solvency capital requirement (SCR)	8.2 bEUR
Solvency ratio	$OF/SCR = 416\,\%$

Table: Solvency II balance sheet: Allianz Lebensversicherung AG (Germany) Solvency and Financial Condition Report (SFCR) 2022; public data: https://www.allianz.de/unternehmen/zahlen-daten-fakten/solvabilitaet-und-finanzlage/

Significant impact on own funds

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FDB

- *FDB* may be larger than own funds
- Model errors in *FDB* calculation can have significant impact on solvency ratio.

Numerical *FDB* calculation

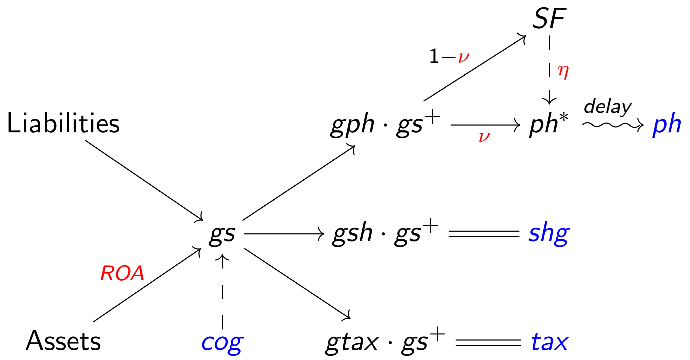
Future discretionary benefits are calculated by Monte Carlo methods:

$$FDB = E\left[\sum_{t=1}^{T} B_t^{-1} ph_t\right]$$

where

- $E[\cdot]$ is the risk neutral expectation;
- $t = 1, \ldots, T$ are (yearly or quarterly or monthly) time steps towards the projection horizon ($\sim$ 60 years);
- B_t^{-1} is the stochastic discount factor associated to an interest rate model;
- ph_t is the policy holder cash flow resulting from profit participation at time steps $1 \leq s < t$;

Profit participation ($gph \cdot gs^+$) vs. profit declaration (ph^*)



- Management rules: Return on assets (UGs, SAA): ROA; Bonus declaration: ν; Surplus fund contribution: η;
- Cash flows: Shareholder gains: shg; Shareholder cost of guarantee: cog; Tax cash flow: tax; Policy holder: ph;

Numerical *FDB* calculation

In order to calculate the *FDB* we need:

- Economic scenario generator (ESG): stochastic scenarios stemming from interest rate, equity and property modules;
- Projection of assets: book values, market values, accounting principles;
- Projection of liabilities: best estimate assumptions for mortality and policy holder behaviour (e.g., surrender);
- Profit and loss (PnL) projection to calculate the gross surplus – the basis for profit participation;
- Management rules: strategic asset allocation (investment targets), realization of unrealized gains and losses, crediting strategy and surplus fund management (profit participation vs. profit declaration!), etc.;

Goal: estimation of FDB

	LP_0	SF_0	UG_0	GB	FDB	$\widehat{FDB}$	$\widehat{LB}$	$\widehat{UB}$
2017	179.40	10.40	41.40	157.37	48.60	?	?	?
2018	190.20	11.00	32.80	161.99	46.20	?	?	?
2019	208.10	11.50	54.00	199.43	47.40	?	?	?
2020	222.87	12.26	66.24	227.56	44.74	?	?	?
2021	233.59	12.50	55.08	231.90	37.61	?	?	?
2022	235.89	11.62	−16.32	167.42	39.50	?	?	?

Table: Allianz Lebensversicherung AG (Germany) Solvency and Financial Condition Report (SFCR) 2022; public data: https://www.allianz.de/unternehmen/zahlen-daten-fakten/solvabilitaet-und-finanzlage/; Values in bEUR;

- Goal: find closed formula to estimate lower and upper bound, $\widehat{LB}$ and $\widehat{UB}$, of FDB from available data;
- If $\varepsilon = (\widehat{UB} - \widehat{LB})/2$ small, estimate $\widehat{FDB} = (\widehat{LB} + \widehat{UB})/2$;

Balance sheet representation of *FDB*

Evolution equation

Let SF_t be the surplus fund at t and DB_t the sum declared benefits at t. Then

$$\Delta(DB_t + SF_t) = DB_t + SF_t - DB_{t-1} - SF_{t-1} = gph \cdot gs_t^+ - ph_t$$

where gph is the constant gross policy holder participation rate, $gs_t^+ = \max(gs_t, 0)$ is the positive gross surplus and ph_t is the policy holder participation cash flow at t.

Balance sheet representation of *FDB*

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Evolution equation

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Integration by parts

$\Delta(f_t g_t) = (\Delta f_t) g_{t-1} + f_t \Delta g_t$ yields, since $\Delta B_t^{-1} = -F_{t-1} B_t^{-1}$:

$$SF_0 - B_T^{-1}(DB_T + SF_T) + \sum B_t^{-1} F_{t-1}(DB_{t-1} + SF_{t-1}) = gph \cdot \sum B_t^{-1} gs_t^+ - \sum B_t^{-1} ph_t$$

Balance sheet representation of *FDB*

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No leakage principle[1]

If $\mathbb{Q}$ is risk-neutral and cash-flows are complete, then ('wealth is conserved'):

$$MV_0 = GB + FDB + SHG - COG + TAX + E_{\mathbb{Q}}[B_T^{-1} MV_T]$$

where

- $MV_0 = LP_0 + SF_0 + UG_0$ initial market value of assets (life provisions plus surplus fund plus unrealized gains);
- $SHG + TAX = (1 - gph) \cdot E[\sum B_t^{-1} gs_t^+]$ are shareholder gains and tax payments;
- and $COG = E[\sum B_t^{-1} gs_t^-]$ are shareholder losses (cost of guarantee).

[1] S. Hochgerner, F. Gach, *Analytical validation formulas for best estimate calculation in traditional life insurance*, Eur. Actuar. J. **9**, pp. 423–443 (2019). https://doi.org/10.1007/s13385-019-00212-2

Balance sheet representation of *FDB*

Theorem (Ref [2])

$$FDB = SF_0 + gph\Big(LP_0 + UG_0 - GB\Big) + gph \cdot COG - I - III$$

where

$$I := E\Big[B_T^{-1}\Big((1-gph)(DB_T + SF_T) + gph(UG_T + LP_T)\Big)\Big]$$

$$III := (1-gph)E\Big[\sum_{t=1}^{T} B_t^{-1} F_{t-1}(DB_{t-1} + SF_{t-1})\Big]$$

$$COG := E\Big[\sum_{t=1}^{T} B_t^{-1} gs_t^{-}\Big]$$

[2] F. Gach & S. Hochgerner, *Estimation of Future Discretionary Benefits in Traditional Life Insurance*, ASTIN Bulletin, 52(3), pp. 835-876 (2022). https://doi.org/10.1017/asb.2022.16

Phenomenology and numerical evidence

Strategy

- Observe that $SF_0 + gph\Big(LP_0 + UG_0 - GB\Big)$ is known at $t = 0$, except for guaranteed benefits, GB, which are purely 'actuarial' (potential caveat: dynamical surrender);
- Make phenomenological assumptions to estimate I, III and COG;
- Verify these assumptions numerically by means of a stochastic asset liability management (sALM) model;[3]
- Apply assumptions to derive estimates $\widehat{LB}$ and $\widehat{UB}$;
- Calculate $\widehat{LB}$ and $\widehat{UB}$ for real data.

[3] F. Gach, S. Hochgerner, E. Kienbacher, G. Schachinger, *Numerical asset liability management modelling in life insurance*, in preparation.

The numerical sALM model

- Asset module:
  - bonds, equity, property, cash;
  - Market values, book values, accounting principles (strict and moderated lower of cost or market principles);
- Liability module:
  - Input: guaranteed (benefit, expense, premium) cash flows;
  - Best estimate assumptions (mortality and surrender) yield contract terminations and discretionary benefit cash flow;
- Management rules:
  - Strategic asset allocation keeps portfolio approx. constant;
  - Negative surplus is avoiding by realizing unrealized gains;
  - Crediting strategy aims to minimize jumps in declaration;
- Economic scenario generator: 5000 scenarios
  - Interest rates: Mean-field Libor market model;[4]
  - Equity and property: geometric Brownian motion;

[4] S. Desmettre, S. Hochgerner, S. Omerovic, S. Thonhauser, *A mean-field extension of the Libor market model*, International J. Theoretical Applied Finance, Vol. **25** No. 01 (2022). https://doi.org/10.1142/S0219024922500054

Run-off assumptions

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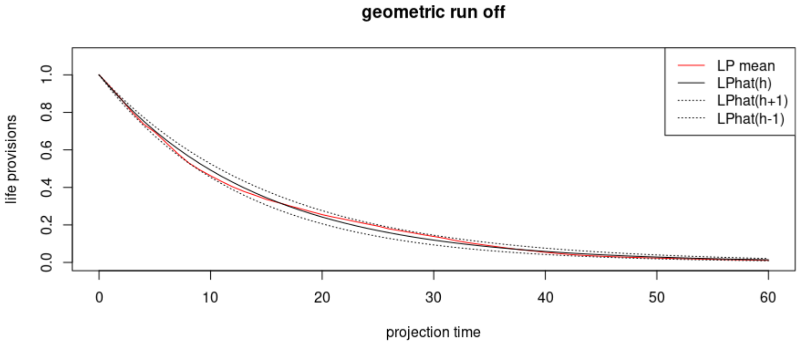
Assumption 1

The projection horizon T corresponds to the run-off time of the liability portfolio such that $SF_T = LP_T = UG_T = 0$.

Assumption 2

The expected life assurance provisions $E[LP_t]$ decrease geometrically: there is a fixed $1 \leq h < T$ such that $E[LP_t] = l_t^h \, LP_0$ where $l_t^h := 2^{-t/h}$ for $t < T$ and $l_T^h := 0$.

Run-off assumptions: numerical evidence



Average declared benefits relation

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Assumption 3

In expectation, the total declared bonuses are a fixed fraction of the life assurance provisions:

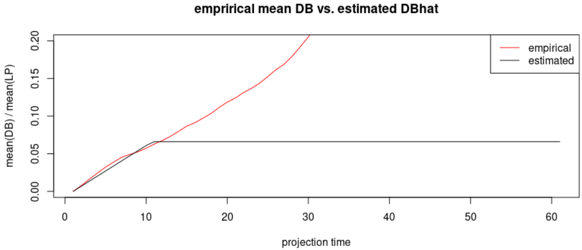
$$E[DB_t^{\leq 0} + DB_t] = \sigma E[LP_t]$$

for all $0 \leq t \leq T$ and a fixed $0 \leq \sigma \leq 1$. Moreover, $E[DB_t^{\leq 0}]$ does not vanish too quickly:

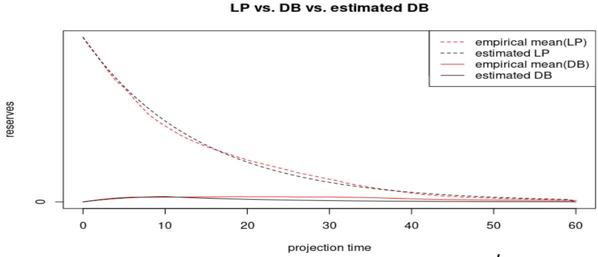
$$E[DB_t] \leq \sigma_t E[LP_t]$$

where $\sigma_t := t\sigma/h$ for $t \leq h$ and $\sigma_t := \sigma$ for $t > h$.

Average declared benefits relation: numerical evidence



Empirical fraction $E[DB_t]/E[LP_t]$ versus estimate $\sigma_t E[LP_t]$;



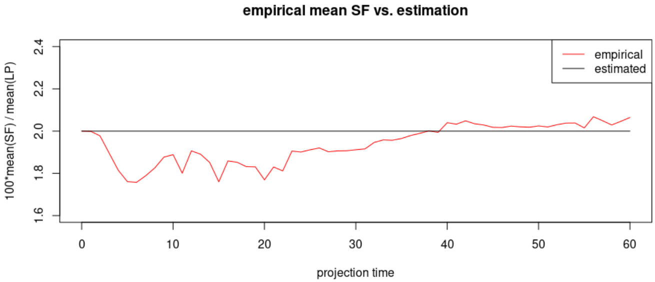
Empirical $E[LP_t]$ and $E[DB_t]$ versus estimated $I_t^h LP_0$ and $\sigma_t I_t^h LP_0$;

Average surplus fund relation

Assumption 4

$E[SF_t] = \vartheta E[LP_t]$ for all $0 \leq t \leq T$.

Numerical evidence



Variation assumption

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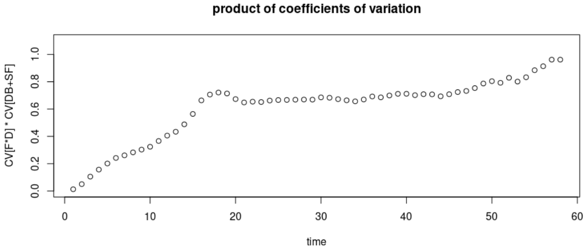
Assumption 5

$CV(F_{t-1}B_t^{-1}) \cdot CV(DB_{t-1} + SF_{t-1}) \leq 1$ for all $1 \leq t \leq T$;

Numerical evidence



Lower bound estimates

These assumptions imply $I \cong \widehat{I}$ and $0 \leq III \leq \widehat{III}$, where

$$\widehat{I} = 0$$

$$\widehat{III} = 2(1 - gph) \sum_{t=1}^{T} \Big(P(0, t-1) - P(0, t) \Big) \Big(\sigma_t + \theta \Big) l_{t-1}^{h} L P_0$$

and $P(0, t)$ is the deterministic discount factor.

Gross surplus

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The gross surplus is given by

$$
\begin{aligned}
gs_t &= \Delta BV_t - \Delta LP_t + cf_t \\
&= F_{t-1}(LP_{t-1} + SF_{t-1}) \\
&\quad + F_{t-1}UG_{t-1} - \Delta UG_t|\mathcal{F}_{t-1} \\
&\quad + ROA_t - ROA_t|\mathcal{F}_{t-1} \\
&\quad - (\rho_t(1-\sigma) - \gamma_t)LP_{t-1}
\end{aligned}
$$

- F_{t-1}: stochastic forward rate;
- $F_{t-1}UG_{t-1} - \Delta UG_t|\mathcal{F}_{t-1}$: contribution due to unrealized gains;
- $ROA_t - ROA_t|\mathcal{F}_{t-1}$: deviation from conditional expectation;
- $(\rho_t(1-\sigma) - \gamma_t)LP_{t-1}$: increment of statutory reserves;

Gross surplus assumptions

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Assumption 6

ρ_t and γ_t are deterministic.

Assumption 7

(1) $ROA_t - E[ROA_t|\mathcal{F}_{t-1}] = 0$;

(2) $F_{t-1}UG_{t-1} - E[\Delta UG_t|\mathcal{F}_{t-1}] \geq P(0,t)^{-1}(I_{t-1}^d - I_t^d)UG_0^{\text{bonds}}$

- where $I_t^d := 2^{-t/d}$ for $t < T$ and $I_T^d := 0$, and d is the duration of the bond portfolio at $t = 0$;
- $UG_0^{\text{bonds}} := \sum_a UG_0^a$ where a runs over all bonds held at $t = 0$;

Gross surplus estimation

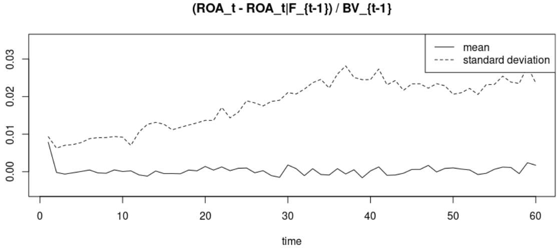
These assumptions imply

$$
\begin{aligned}
gs_t &\geq \widehat{gs}_t \\
&:= F_{t-1}E[BV_{t-1}] + P(0,t)^{-1}(I^d_{t-1} - I^d_t)UG^b_0 - \rho_t V_{t-1} + \gamma_t LP_{t-1} \\
&= \left((1+\vartheta)F_{t-1} + \frac{I^d_{t-1} - I^d_t}{P(0,t)I^h_{t-1}} \frac{UG^b_0}{LP_0} - (1-\sigma)\rho_t + \gamma_t\right) I^h_{t-1} LP_0
\end{aligned}
$$

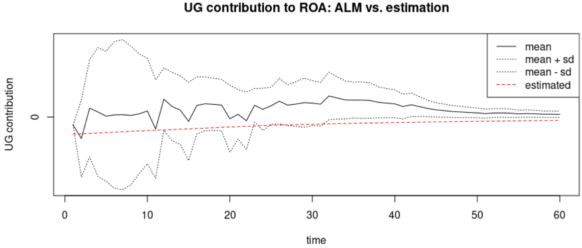
where only F_{t-1} is stochastic.

Gross surplus assumptions: numerical evidence

Predictabilty of *ROA*

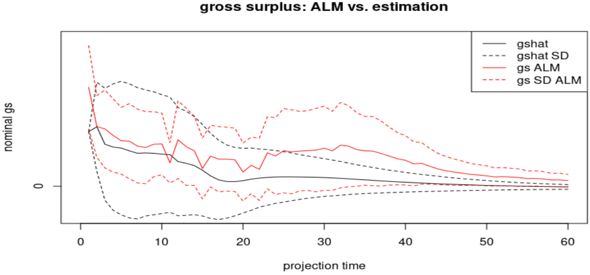


Contribution of unrealized gains

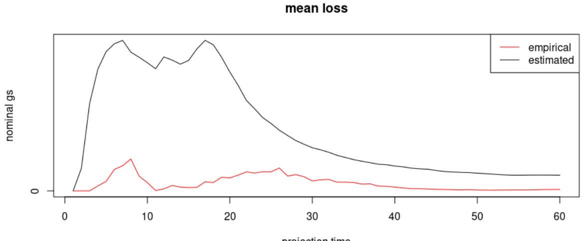


$\widehat{gs}$: a posteriori numerical evidence

Gross surplus: sALM model vs estimation



Comparison of mean loss



Estimating cost of guarantees

With these assumptions, $0 \leq COG \leq \widehat{COG}$:

$$\widehat{COG} = E\left[\sum_{t=1}^{T} B_t^{-1} \widehat{gs}_t^-\right] = \sum_{t=1}^{T} \mathcal{O}_t^- I_{t-1}^h LP_0$$

where

$$\mathcal{O}_t^- := E\left[B_t^{-1}\left((1+\vartheta)F_{t-1} + \frac{I_{t-1}^d - I_t^d}{P(0,t)I_{t-1}^h}\frac{UG_0^b}{LP_0} - (1-\sigma)\rho_t + \gamma_t\right)^-\right]$$

is the value at 0 of the floorlet (i.e., negative caplet) at $t = 0$, and this can be calculated using Black's formula from market data:

- current interest rate curve (EIOPA);
- caplet implied volatilities;

Estimates: lower and upper bounds

Lower and upper bounds, $\widehat{LB} \leq FDB \leq \widehat{UB}$ where

$$\widehat{LB} := SF_0 + gph\Big(LP_0 + UG_0 - GB\Big) - \widehat{III}$$
$$\widehat{UB} := SF_0 + gph\Big(LP_0 + UG_0 - GB\Big) + gph \cdot \widehat{COG}$$

If $\varepsilon := (\widehat{UB} - \widehat{LB})/2$ is sufficiently small (e.g., in comparison to MV_0), then

$$\widehat{FDB} = \frac{\widehat{LB} + \widehat{UB}}{2} = SF_0 + gph\Big(LP_0 + UG_0 - GB\Big) + \frac{gph \cdot \widehat{COG} - \widehat{III}}{2}$$

may be used as a reasonable estimator for FDB, with error bounds $\widehat{FDB} \pm \varepsilon$.

Application to public data: Allianz LV AG

	LP_0	SF_0	UG_0	GB	FDB	$\widehat{FDB}$	$\widehat{LB}$	$\widehat{UB}$
2017	179.40	10.40	41.40	157.37	48.60	46.30	43.43	49.17
2018	190.20	11.00	32.80	161.99	46.20	44.36	41.32	47.39
2019	208.10	11.50	54.00	199.43	47.40	45.99	43.49	48.48
2020	222.87	12.26	66.24	227.56	44.74	48.71	43.25	54.17
2021	233.59	12.50	55.08	231.90	37.61	44.10	38.90	49.29
2022	235.89	11.62	−16.32	167.42	39.50	39.42	32.40	46.45

Table: Allianz LV AG; quantities in bEUR;

	FDB	$\widehat{FDB}$	$\widehat{LB}$	$\widehat{UB}$	$\varepsilon=(\widehat{UB}-\widehat{LB})/2$	$\delta=\widehat{FDB}-FDB$
2017	21.02	20.03	18.78	21.27	1.24	−1.00
2018	19.74	18.96	17.66	20.25	1.30	−0.79
2019	17.32	16.81	15.90	17.72	0.91	−0.52
2020	14.85	16.16	14.35	17.97	1.81	1.32
2021	12.49	14.64	12.92	16.37	1.72	2.15
2022	17.08	17.05	14.01	20.09	3.04	−0.03

Table: Allianz LV AG; quantities in percent of $MV_0 = LP_0 + SF_0 + UG_0$;

Public and anonymized data

	FDB	$\widehat{FDB}$	$\widehat{LB}$	$\widehat{UB}$	ε	δ
Allianz 2017	100.00	95.27	89.36	101.17	5.90	−4.73
Allianz 2018	100.00	96.01	89.44	102.57	6.56	−3.99
Allianz 2019	100.00	97.02	91.76	102.28	5.26	−2.98
Allianz 2020	100.00	108.87	96.68	121.07	12.20	8.87
Allianz 2021	100.00	117.25	103.44	131.06	13.81	17.25
Allianz 2022	100.00	99.81	82.03	117.59	17.78	−0.19
A 2018	100.00	125.10	94.59	155.60	30.50	25.10
B 2019	100.00	84.96	63.78	106.14	21.18	−15.04
C 2019	100.00	110.02	77.05	142.99	32.97	10.02
D 2022	100.00	101.32	90.09	112.55	11.23	1.32
E 2022	100.00	96.51	89.40	103.62	7.11	−3.49

Table: Results in percent of FDB. Estimation is considered successful if $|\delta| = |\widehat{FDB} - FDB| \leq \varepsilon = (\widehat{UB} - \widehat{LB})/2$.

Conclusions

- Phenomenological assumptions imply algebraic formulas to estimate lower and upper bounds, and *FDB* as the mean value of these.
- The assumptions are verified by means of a stochastic ALM model. This does *not* prove the general validity of the assumptions.
- Potential applications:
  - Quick and easy validation of sALM model for companies, auditors, supervisors . . . ;
  - Sensitivity analysis of portfolio;
  - Generate ALM scenarios of Solvency II balance sheet;
  - . . .
- The estimation formula is *not* a substitute for a stochastic best estimate calculation.
- In our experience, the formula gives reasonable and useful results (cf. previous slides), but there is certainly room for improvements.